



Trajectories of energetic particles in the magnetosphere

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1. Classification of particle trajectories in magnetosphere.

- Large variety, depending on particle energy and on magnetic and electric fields.
- Energies: from < 1eV (ionospheric plasma) to > 10 17 eV (cosmic rays)
- Classification (of particles) according to characteristic dimensions of their trajectories (d) with respect to dimension of magnetosphere (d_m) :

 $d^{3} d_{m} \qquad d \ll d_{m}$

Equation of motion (particle with rest mass m, charge q, velocity **v** in static magnetic field **B**):

(1)
$$d(\gamma.m.\mathbf{v})/dt = d(\mathbf{p})/dt = q \cdot \mathbf{v} \times \mathbf{B}$$
 / $\cdot \mathbf{p}$ ($\mathbf{p} \parallel \mathbf{v}$)
 $\partial \mathbf{B}/\partial t = 0$ (-rot (\mathbf{E})) $d(\mathbf{p})^2/dt = 0$
 $\mathbf{p} = |\mathbf{p}|$ is constant of motion
 $\mathbf{v} = d\mathbf{x}/dt = c^2 \mathbf{p} / U$
Total energy:
 $U = (c^2.p^2 + m^2.c^4)^{1/2}$ $U = \gamma m c^2$ $\gamma = (1-v^2/c^2)^{-1/2}$
 $U, |\mathbf{v}|$ and γ are constants of motion (in static \mathbf{B})

Dipolar magnetic field

B (**r**) = - grad (
$$\mu_0$$
 . **M** . **r** / (4 π r³))

r – radiusvector (from center of dipole), $\mu_0=4\pi$.10⁻⁷ (SI)

Since **B** is static, the operator d/dt can be replaced by (**v** . \bigtriangledown) and (1) has form

(2)

$$m . (\mathbf{v} . \nabla) \mathbf{v} = q. \mathbf{v} \times \mathbf{B}$$
(3)

Eq. (3) with **B** (2) can be rewritten in non-dimensional form by using the Stormer length as a unit of length

Stormer length

 $I_s = (\mu_o . M . |q| / (4 \pi m v))^{1/2}$

For Earth : $M_E = 8.1 \cdot 10^{22} \text{ A.m}^2$

Physical meaning of I_s: Curvature radius **R** of any smooth curve is $R/R^2 = (t \cdot \nabla)t$ (4)**t** – unit tangent vector to the curve at given point $(\mathbf{v} = d\mathbf{r}/dt)$ $\mathbf{t} = \mathbf{v}/\mathbf{v}$ RHS of (4) is equivalent to $[(\mathbf{v}/\mathbf{v}) \cdot \nabla](\mathbf{v}/\mathbf{v})$ Using (2 and 3) $\mathbf{R}/\mathbf{R}^2 = -(\mu_0 \mathbf{q} / (4 \pi \mathbf{m} \mathbf{v}^2)) \mathbf{v} \mathbf{x} \nabla (\mathbf{M} \cdot \mathbf{r}/\mathbf{r}^3)$ (5)Estimate of R from (5) is according [1] R = r $(r/I_s)^2$ F, where F is geometric coefficient of order of unity (depending on angle between **B** and **v**) (6)

Meaning of (6) : $R = r (r/l_s)^2 F$

- If $I_s > r$, curvature radius (R) is significantly smaller than r If $I_s < r$, R > r
- Characteristic distance (r) is size of magnetosphere

Example (for students). Compute the Stormer length of

a. electron with kinetic energy 1 eV, 1 MeV, 10 GeV

b. proton with these energies

in Earth magnetosphere and compare the values with distance to the sub-solar point of the magnetosphere (~10 Re).

How these lengths are changed for Jupiter ($M_J = 20.000 M_F$,

 $1 R_{J} = 71.400 \text{ km}$, sub-solar point at $50 - 100 R_{J}$?

Two categories of particles (their trajectories) in the magnetosphere of Earth:

 a. Particles for which the Stormer length is comparable or smaller than dimensions of magnetosphere. For protons of energy above ~10 MeV (kinetic), for electrons above ~100 MeV.

Cosmic rays (galactic, anomalous, solar and accelerated in the interplanetary space to those energies) – Stormer trajectories (part 3).

b. Particles with *Stormer length substantially larger than size of magnetosphere*.

Trapped particles in radiation belts, auroral particles, particles of magnetospheric plasma populations - guiding center approach (part 2).

2. Low energy particles: short summary

If $I_s >> d_m$, theory of perturbation is used [1,2,4]. Magnetic and electric fields are "given", not influenced by particle motion.

2.1. Perturbation theory.

2.1.1. Homogeneous (unperturbed) magnetic field

Rotation with gyroradius ($r_c = m.v_{\perp}/(|q|.B)$), gyrofrequency ($\omega_c = |q|.B / m$). Magnetic flux in Larmor ring (Φ) is proportional to the diamagnetic moment of particle motion (μ)

$$(\mu = \pi \cdot r_c^2 \cdot J = m \cdot v_{\perp}^2 / 2B = W_{\perp} / B; J = |q| \cdot \omega_c / 2\pi; W - kinetic energe)$$

$$\Phi = \pi \cdot r_c^2 \cdot B = 2 \cdot \pi \cdot \mu \cdot m / q^2$$
(7)

<u>**Collisions.</u>** Change of velocity is $\delta v = 1/m \, . \int F_i \, dt$, F_i is interaction force, integration over time of interaction. Gyration center is shifted.</u>

If the force has a predominant direction (average F - 0), drift appears $v_F = (F \times B)/(q.B^2)$ (8) **Presence of electric field E -0.** Reference system S' with **E'** = 0 is found. Its velocity with respect to S ($v_{E\perp} = (E \times B)/(B^2)$) is superimposed on rotation motion.

<u>Slow variations of magnetic field (relative variation of B is small</u> <u>over one gyroperiod):</u> Change of energy over gyroperiod is [1]

 $\Delta W_{\perp} = W_{\perp} . (\Delta B / B) \qquad \Rightarrow \qquad \mu = W_{\perp} / B = \text{const}; \qquad (9)$

μ is saved if temporal and/or spatial variations of magnetic and electric field met by the gyrating particle over one gyroperiod are small (*not valid for high energy particles – cosmic rays).*

2.1.2. Drift motion [4].

Guiding center system GCS

(if at any instant of time we can define a (moving) **frame of reference in which an observer sees the particle in a periodic orbit** perpendicular to magnetic field (with single periodicity and during at least one full cycle)).

Order of drifts: by value of ($r_c . |\nabla B|/B$).

Zero order drifts: uniform static **B** plus external non-magnetic force **F** which is constant in time and space. To find GCS: perpendicular velocity (v_F) of the reference system is found in which the external force $F_{\perp} * (= F_{\perp}, * \text{ are quantities in GCS})$ is balanced by the induced electric field (q . E* = q . $v_F \times B$) Similar to (8) $v_F = (F \times B)/(q.B^2)$

First order drifts: *non-uniform* **B.** Gradients of B cause 1-st order drifts which are *energy dependent*.

Combined gradient-curvature drift

$$\mathbf{v}_{CG} = (m/2.q.B^2).(v_{\perp}^2 + 2.v_{\parallel}^2) \mathbf{e} \times \nabla_{\perp} B =$$

= m.v²/(2.q.B²).(1+cos² α) **e** x ∇_{\perp} B (**e** is unit vector along **B**, α PA) (10)

If $r_c |\nabla B|/B <<1 \Rightarrow v_{CG} << v$

Second order drifts: time-dependent zero or first order drifts (e.g. timedependent **E**, or direction-changing drift along a curved equipotential). If frame of reference moves with **v** (*changing in time*), it is accelerated and *inertial force appears* to an observer in that frame $-m \cdot dv/dt \Rightarrow$ "true" GCS is moving with velocity **v** plus additional drift (**v**_s) :

$$\mathbf{v}_{s} = - (m/qB) \cdot (d\mathbf{v}/dt \times \mathbf{e})$$
(11)

which is necessary to induce electric field that cancels the effect of inertial force in GCS. For GCS approximation to remain valid, the change of **v** during one gyroperiod must be very small

(if
$$T_c.(dv/dt)/v \ll 1$$
, (11) implies $v_s \ll v$).

Combining (11, 10, 8) more general expression for drift is:

 $\mathbf{v}_{\rm D} = \mathbf{e}/(q.B) \times [-\mathbf{F} + (m/2B).(v_{\perp}^2 + 2v_{\parallel}^2) \nabla_{\perp}B + m.d\mathbf{v}_{\rm D}/dt]$ (12)

F represents all external forces, including all electric field forces (electrostatic and induced). For computations of v_D the iterations are needed. Complete expression for v_D including higher orders is in [7].

2.2. Adiabatic invariants.

If particle motion is *possible to assume as superposition of more* (3 in case of geomagnetic trapped population) *cyclic motions* (with different periodicities), the useful approach to trajectory checking is by *adiabatic invariants* connected with these motions. Theory is e.g. in [9].

First invariant. Gyroradius is defined in GCS: $r_c = p_{\perp}*/q.B*$; the associated gyroperiod $T_c = (2\pi . r_c)/v_{\perp}*= (2\pi . m*)/q.B*$ and the gyrofrequency $\omega_c = 2\pi/T_c = q.B*/m*$. These expressions are valid relativistically assuming $m = m_o.\gamma$, $\gamma = (1-\beta^2)^{-1/2}$; $\beta = v*/c$.

When either a zero order or first order drift acts alone, the kinetic energy of particle is conserved in original frame of reference (OFR). When both drifts act together, kinetic energy does not remain constant.

There is, however, *quantity (related to gyration) conserved when GCS approximation holds*, that is, as long as a spatial variations are very small over T_c and time variations are very small over T_c .

 $\mu = p_{\perp}^{*2}/(2m_{o}B) = \text{const} (p_{\perp}^{*} = p^{*} \text{ is particle momentum in GCS}) (13)$ Relativistic magnetic moment or 1st adiabatic invariant.

Second invariant.

Related to the *bounce motion* (between the mirror points, for trapped particles in the magnetic mirror). $\mathbf{p} = m.\mathbf{v} + e.\mathbf{A}$

(A is magnetic field vector potential)

The quantity

J =∮p_{II}ds

(14)

is conserved (the path of integration is *along a field-line*: from the equator to the upper mirror point, back along the field-line to the lower mirror point, and then back to the equator).

Third invariant.

is associated with the *cyclic longitudinal drift of particles* around the earth. Similarly to guiding center associated with a particle gyromotion around field-lines, the *bounce center lies on the equatorial plane, when particles are azimuthally drifting around the earth*.

$$\mathsf{K} = \mathsf{e} \cdot \oint \mathsf{A}_{\phi} \, \mathsf{ds} = \mathsf{e} \cdot \Phi$$

Where A_{ϕ} is magnetic vector potential (its ϕ component). *Integration is along the guiding drift shell*. Φ is the total magnetic flux enclosed by the drift trajectory (by the orbit of the bounce centre around the earth).

(15)

- Adiabatic approximation holds for $T_c \ll T_{bounce} \ll T_{drift}$.
- Periodic forces on time scales T_c , T_{bounce} , T_{drift} may yield in violation of corresponding invariants.

Basic Components of Particle Motion: bounce, gyration and drift



Three cyclic motions, each one connected with adiabatic invariant



Frequencies of cyclic motions for equatorially trapped charged particles.

Copied from [10].

Upper right corner: adiabatic approach is not valid (periodicieties are comparable) Cosmic rays

Fig. 6. Contours of constant adiabatic gyration, bounce, and drift frequency for equatorially mirroring particles in a dipole field. Adiabatic approximation fails in upper right-hand corners ($E \sim 1 \text{ GeV}$, $L \sim 8$), since $\Omega_1 \sim \Omega_2 \sim \Omega_3$ implies $|\varepsilon| \sim 1$.

3. High energy particles

3.1. Stormer theory for dipolar field.

$$B(\mathbf{r},\lambda) = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3} \sqrt{1 + 3\sin^2 \lambda}$$

 λ – magnetic latitude ψ -angle (z,r, in Rz plane)



Equation of motion (m.dv/dt=q.v x B) is solved as (a) *motion of meridian plane* and (b) *motion within the meridian plane*.

For (a) using equivalence of moment of force and time derivative of the moment of momentum with respect to z axis ($M=RF_{\phi}=d/dt(Rm.v_{\phi})$) [11]

 $(RF_{\phi} = R^2.m.d\phi/dt)$

Since v is constant, time is substituted by geometrical variable s = v.t.

 $R^2.d\phi/ds = I_s^2.(R^2/r^3) + b$, where I_s is Stormer length (A)

Adjusting the scale as $I_s = 1$, a universal equation is obtained independent on mass and velocity of particle.

- Meaning of I_s in equatorial plane of dipolar field:
- Stable trajectory: Centrifugal force = Lorentz force
- $m.v^2/R_o = q.v.\mu_o.M/(4.\pi.R_o^3) \implies R_o = (\mu_o.M.|q|/(4.\pi.m.v))^{1/2} = I_s$

Circular periodic orbit of particle in dipolar field is that at I_s .

Limit of (A) for $r \rightarrow \infty$ is moment of particle momentum with respect to z outside the influence of dipolar field. By putting $M \mid_{r \rightarrow \infty} = -2.\gamma$, the form of motion of meridian plane (with the particle) is

$$R^{2}.d\phi/ds = \pm (R^{2}/r^{3}) - 2.\gamma \qquad (\pm \text{ for sign of charge}) \qquad (B)$$

For (b) (motion *in* meridian plane, (Rz)). Kinetic energy (T=1/2.m.v²) is conserved. Using coordinate system with m = v = 1,

(D)

$$T = (1/2) [R^2 (d\phi/ds)^2 + (dr/ds)^2 + r^2 (d\psi/ds)^2] = 1/2.$$
 (C)

Motion is divided into (1) parallel and (2) perpendicular to meridian. Kinetic energy in Rz plane is Q'.

According to (C,B) \Rightarrow Q' = (1/2).[1-(±(R/r^3) - 2.\gamma/R)^2]

- From (D): $Q' = (1/2) [1 (\pm (R/r^3) 2.\gamma/R)^2]$ (D)
- 1. Motion in meridian plane is independent on rotation of that plane (Q' is indepentent on ϕ).
- 2. For given γ the kinetic energy Q' is only function of coordinates.

For Q = 2Q' there exist 2 characteristic values:

maximum (Q=2, T=1). *Particle is moving in meridian plane*.

minimum (Q=0). Particle is moving perpendicular to meridian plane and velocity projection to Rz plane changes its sign (particle after reaching the point Q=0 is moving in reversed direction). *The line Q=0 at plane Rz is enclosing the region forbidden for particle.*

(D) Provides system of equipotential lines. *Q=0 is most important*. By rotating *Q=0 around z axis, the rotational surface is obtained:* segment of space near the dipole to which the particles cannot enter. Shape and extent of this segment of space depends on γ.

3.2. Allowed and forbidden trajectories.

When using system m = v = 1 then Q = v_{mer}^2 , $v_n = \pm (1-Q)^{\frac{1}{2}}$ (meridional and normal components of v).

Adjusting ω as angle between instantaneous velocity vector v of the particle and normal to meridian plane which follows the particle in its orbit (measured consistently with ϕ , i.e. from west to east)

$$v_n^2 = v^2 \cdot \cos^2(\omega) = \cos^2(\omega) = 1-Q$$

According to (D) for positively charged particles

$$\cos(\omega) = (\mathsf{R}/\mathsf{r}^3) - (2.\gamma/\mathsf{R}) \tag{E}$$

$$\cos(\lambda) = R/r \implies \cos(\omega) = \cos(\lambda)/r^2 - 2.\gamma/(r.\cos(\lambda))$$
 (F)

 λ is latitude.

Solving (F):
$$r = (\cos^2(\lambda)/(\gamma \pm (\gamma^2 + \cos(\omega).\cos^3(\lambda))^{1/2})$$
 (G)

(H)

For limiting surface Q=0, $cos(\omega)$ = ± 1 and

 $r = \cos^2(\lambda) / (\gamma \pm (\gamma^2 \pm \cos^3(\lambda))^{1/2})$

The solutions of : $r = \cos^2(\lambda)/(\gamma \pm (\gamma^2 \pm \cos^3(\lambda))^{1/2})$

$$\mathbf{r}_1 = \cos^2(\lambda) / (\gamma + (\gamma^2 + \cos^3(\lambda))^{\frac{1}{2}})$$

 $\mathsf{r}_2 = \cos^2(\lambda)/(\gamma + (\gamma^2 - \cos^3(\lambda))^{\frac{1}{2}})$

$$r_3 = \cos^2(\lambda)/(\gamma - (\gamma^2 - \cos^3(\lambda))^{\frac{1}{2}})$$

3 solutions for $\gamma^2 > \cos^3(\lambda)$ (a)

1 solution for $\gamma^2 < \cos^3(\lambda)$ (b)

In case (a) the radial line crosses surface Q=0 three times (*region of allowed trajectories Q>0 is composed of two isolated regions*);

In case (b) there is only one region of allowed trajectories.

Max. $\cos^{3}(\lambda) = 1$, for $\gamma > 1$ two regions of allowed trajectories

Details and complete calculations regarding the problem "given a particle approaching a magnetic dipole from infinity, what is its trajectory?" is e.g. in book [13]. *No general solution to the problem, but it is possible to define permitted and forbidden zones for any particle approaching from infinity.*



Specifying particle momentum – the Earth can be drawn in these diagrams. For γ =1 there is a critical momentum at which Stormer length is equal to radius of Earth (p₁=59.6GeV/c). For this momentum the Earth is a circle with r₁=1. For p=4p₁ the r=2.

If $\gamma > 1$, particles cannot come to r < 1

Using (F) - interval of angles ω < $\omega_{\rm o}$ with forbidden directions is obtained

("opened" for $\pi > \omega > \omega_0$ measured from east direction)

$$\cos(\omega_{o}) = \cos(\lambda)/r^{2} - 2.\gamma/(r.\cos(\lambda)) \qquad (r < 1)$$

Stormer cone

Access of positively charged particles is "more difficult" from east.

For given position and given momentum there is a cone with axis in east direction for which the access of particles is forbidden.

Its extent is decreasing with increase of momentum and with latitude.



FIG. 5. Størmer's forbidden cone, and the shadow cone.

Shadow cone is due to the presence of earth body. (from A.M. Hillas, Cosmic Rays, 1972)

3.3. Concept of cut-off rigidity and asymptotic directions.

Usually the effects of geomagnetic field on cosmic rays are expressed in terms of <u>rigidity</u>.



$$\frac{p^{2}c^{2} = -m_{0}^{2}c^{\dagger} + \mathcal{U}^{2} \longrightarrow \mathcal{U}^{2} = p^{2}c^{2} + m_{0}^{2}c^{\dagger}}{\mathcal{U} = E_{0} + E_{kin}} \xrightarrow{E_{0} + E_{kin}} = E_{kin} = (g^{2} - 1) \cdot E_{0}$$

$$\frac{\mathcal{U} = E_{0} + E_{kin}}{\mathcal{V}m_{0}c^{2}} = m_{0}c^{2} + E_{kin} \xrightarrow{E_{kin}} = (g^{2} - 1) \cdot E_{0}$$
(16)
$$\frac{\mathcal{R} = \frac{p^{2}}{\mathcal{Z}}}{\mathcal{R} = \frac{p^{2}}{\mathcal{Z}}} = 2 \xrightarrow{\mathcal{U}^{2}c^{2}} = (g^{2} - 1) \cdot \frac{m_{0}^{2}c^{\dagger}}{\mathcal{Z}^{2}} \xrightarrow{\mathcal{C}^{4}} = (1 + 2\frac{E_{k}}{\mathcal{E}_{0}} + \frac{E_{k}^{2}}{\mathcal{E}_{0}^{2}} \xrightarrow{\mathcal{C}^{4}}) \xrightarrow{\mathcal{M}_{0}^{2}c^{\dagger}} = \left(\frac{\mathcal{L}E_{k}E_{0} + E_{k}^{2}}{\mathcal{E}_{0}^{2}} \xrightarrow{\mathcal{L}^{2}} \xrightarrow{\mathcal{L}^{2}} = \left(\frac{\mathcal{L}E_{k}E_{0} + E_{k}^{2}}{\mathcal{E}_{0}^{2}} \xrightarrow{\mathcal{L}^{2}} \xrightarrow{\mathcal$$

 $R = \frac{A}{2} \left[T_{\mu} \cdot (T_{\mu} + 2E_{op}) \right]$ GV,



R/00 -> r/00

For given B the gyroradius is proportional to rigidity

 $(9,109,10^{-31} \text{ kg})$ $(1,673,10^{-27} \text{ kg})$ $(1.675,10^{27} \text{ kg})$

Directional definitions for cosmic ray access (from Cooke, D.J. et al., Il Nuovo Cimento, 14 C, N 3, 213-233, 1991):

<u>Cones</u> (each definition is for charged particles of a *single specified rigidity* value arriving at *particular point* in geomagnetic field):

<u>Allowed cone</u> – solid angle with all directions of arrival of all trajectories which do not intersect the earth and which cannot possess sections asymptotic to bound periodic orbits (because rigidity is too high to permit such sections to exist in the directions of arrival concerned).

<u>Main cone</u> – The boundary of allowed cone. It is composed of trajectories which are asymptotic to the simplest bound periodic orbits and trajectories which graze the surface of earth (For this purpose the surface of earth is generally taken to be at the top of effective atmosphere).

Forbidden cone – The solid angle region within which all directions of arrival correspond to *trajectories which, in absence of solid earth, would be permanently bound in the geomagnetic field*. Access in these directions from outside the field is therefore impossible.

<u>Stormer cone</u> – the boundary of forbidden cone. In axially symmetric field the surface forms a right circular cone.

<u>Shadow cone</u> – the solid angle containing all directions of particle arrival which are excluded due to short range earth intersections of the approaching trajectories, while the particle loops around the local field lines

<u>Penumbra</u> – the solid angle region contained between the main cone and the Stormer cone. It contains a complex structure of allowed and forbidden bands of arrival directions.

<u>Rigidity picture definitions</u> (each definition refers to particles *arriving at a particular site* within geomagnetic field *from a specified direction*.

<u>Cut-off rigidity</u> – location of transition, in rigidity space, from allowed to forbidden trajectories as rigidity is decreased

<u>Stormer cut-off rigidity</u>, R_s – the rigidity value for which the Stormer cone lies in the given direction. In a dipole field *the direct access for particles of all rigidity values lower than Stormer cut-off rigidity is forbidden from outside the field*. In a dipole approximation (in GV)

$$R_{s} = (M.\cos^{4}(\lambda)) / \{r^{2} \cdot [1 + (1 - \cos^{3}(\lambda) \cdot \cos(\varepsilon) \cdot \sin(\xi))^{1/2}]^{2} \}$$
(18)

- M is the dipole moment and has a normalized value of 59.6 when r is expressed in units of earth radii
- $\boldsymbol{\lambda}$ is magnetic latitude
- r is distance from the dipole in earth radii
- ϵ is azimuth angle measured clockwise from geomagnetic east direction (for positive particles)
- $\boldsymbol{\xi}$ is the angle from the local magnetic zenith direction

<u>Upper cut-off rigidity (R</u>U) – rigidity value of the *highest* detected allowed/forbidden transition among a set of computed trajectories.

<u>Lower cut-off rigidity (R_L)</u> – the lowest detected cut-off value (i.e. the rigidity value of the *lowest allowed/forbidden transitions* observed in a set of computed trajectories). If no penumbra exists, $R_L = R_U$.

Effective cut-off rigidity (R_E) – the total effect of penumbral structure in a given direction may be represented usefully by this single value. Effective cut-offs may be either linear averages of the allowed rigidity intervals in penumbra, or functions weighted for cosmic ray spectrum and/or detector response. For linear weighting

R_{U}

 $R_{E} = R_{U} - \sum \Delta R_{i} \text{ (allowed)}$ R_{L} (19)

Where the trajectory calculations are performed at rigidity interval (step) $\Delta R_{i}.$

Example (students):

What is the Stormer cut-off rigidity for cosmic ray access to Sofia position

a. from the vertical direction, b. from direction 30° declined from zenith from east (magnetic) and c. from west (magnetic). What kinetic energy of protons and of anomalous cosmic ray oxygen ions (A=16, Z=1) these values correspond

Lat = 42.45N, Long = 23.20E

http://swdcwww.kugi.kyoto-u.ac.jp/igrf/gggm/index.html

Conversion from Geographic to Geomagnetic coordinates Geomagnetic: 40.75 N104.00 E

Asymptotic directions.



Allowed trajectory. Charged particle of cosmic rays arriving above the site of particular station from a given direction (local) arrived formerly to magnetospheric boundary at another direction (asymptotic direction). Figure (*from Shea,M.A. and Smart, D.F., ERP No 524, AFCRL-TR-75-0381, 1975*) illustrates the definition of *asymptotic latitude and longitude*.

The "asymptotic latitude", Λ , is given by

$$\tan \Lambda = \frac{-\mathbf{v}_{\theta} \sin \theta + \mathbf{v}_{r} \cos \theta}{\left[\mathbf{v}_{\theta}^{2} + \left(\mathbf{v}_{\phi} \cos \theta + \mathbf{v}_{r} \sin \theta^{2}\right)\right]^{1/2}}$$

and the "asymptotic longitude", ψ , by

$$\psi = \phi + \arctan\left(\frac{v_{\phi}}{v_{\theta}\cos\theta + v_{r}\sin\theta}\right)$$
(20)

where

 θ = co-latitude

 ϕ = longitude measured eastward from the Greenwich meridian, and v_r, v_θ, v_ϕ = velocity components in r, θ , and ϕ directions.



Asymptotic directions are usually plotted as (long, lat) projection on the map. The computed asymptotic longitudes and latitudes for a high cut-off rigidity station and vertical access. Wide range of asymptotic longitudes for 13-20 GV. Example for *high cut-off rigidity* station (*from Shea, M.A. and D.F. Smart, 1975*)

ASYMPTOTIC LONGITUDE



Figure A12. Station Orientated World Map Illustrating the Asymptotic Directions for Churchill, Canada

Asymptotic directions computed for a *low cut-off rigidity station*. The interval of asymptotic longitudes is narrower. (*from Shea, M.A. and D.F. Smart, 1975*)

CHURCHILL, CANADA

GEOGRAPHIC LATITUDE = 58.75 GEOGRAPHIC LONGITUDE = 265.91

EFFECTIVE VERTICAL CUTOFF RIGID JTY = .21 GV

RIGIDITY (Gy)	A SYMP	TOTIC Long.
20.00	4.3	- 77
19.00	41	+72
18.00	60	-72
17.00	39	-73
16.00	38	-73
15.00	37	-74
14.00	36	-74
13.00	35	-75
12.00	34	-75
11.00	34	-76
10.00	33	-76
9.00	33	-76
6.00	32	-75
7.00	30	- / 4
6.00	26	- 74
5.00	23	- 7 3
9.00	16	-73
2.00	7	-71
1.90	6	-71
1.80	5	-71
1.70	4	-70
1.60	2	-69
1.50	1	- 69
1.40	-1	-68
1.30	-2	-68
1.20		- 66
1.10	~6	- 65
1.00	- 1 2	- 6 7
. 90	- 15	- 60
. 70	-19	- 56
.60	- 23	- 51
. 50	-28	-43
. 40	- 31	-26
. 39	- 31	- 24
. 38	- 31	-21
. 37	- 31	-19
.36	- 31	-16
. 35	-39	-13
. 34	- 29	-9
. 3 3	- 20	- 2
. 32	- 25	2
. 31	- 27	6
. 30	- 19	11
- 24	- 14	16
.27	-9	22
.26	-1	29
.25	9	38
. 24	23	53
. 23	32	90
• 22	- 28	384
. 21	13	379

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